Active matter – Final project
DRSTP Advanced Topics in Theoretical Physics (Spring 2025)

Yann-Edwin Keta
keta@lorentz.leidenuniv.nl

You can choose either the computational or the analytical project. Please submit your derivations and/or scripts by email before Monday 9th June. Questions are welcome.

I. COMPUTATIONAL PROJECT

Consider an ensemble of N particles on a one-dimensional periodic ring, with positions

$$r_i = r_i^0 + u_i, \tag{1}$$
$$r_i^0 = i\,a, \tag{2}$$
$$r_{i+N} = r_i, \tag{3}$$

where a is the effective particle diameter, and moving according to the following equation of motion,

$$\zeta \dot{u}_i = B(u_{i+1} + u_{i-1} - 2u_i) + \lambda_i^{\mathrm{act}}, \tag{4}$$

where ζ is a drag coefficient, B is akin to an elastic constant, and λ_i^{act} is a stochastic force following an Orsntein-Uhlenbeck process,

$$\tau_p \dot{\lambda}_i = -\lambda_i + \sqrt{2\tau_p f^2}\,\eta_i, \tag{5}$$

where τ_p is a persistence time, f is a force scale, and η_i is a Gaussian white noise with mean $\langle \eta_i(t) \rangle = 0$ and variance $\langle \eta_i(t)\eta_j(t') \rangle = \delta_{ij}\,\delta(t-t')$.

Q: How does the time-averaged displacement variance $\frac{1}{N}\sum_{i=1}^{N}(u_i(t) - \overline{u}(t))^2$, where $\overline{u}(t) = \frac{1}{N}\sum_{i=1}^{N} u_i(t)$ is the instantaneous average displacement, depend on N for large N *in steady state* and for all other parameters constant?

II. ANALYTICAL PROJECT

Consider the following equation of motion for the displacement field $u(x,t)$ of a one-dimensional continuous and infinite elastic system,

$$\zeta \frac{\partial}{\partial t} u(x,t) = \eta \frac{\partial^2}{\partial x^2}\frac{\partial}{\partial t} u(x,t) + B \frac{\partial^2}{\partial x^2} u(x,t) + \lambda^{\mathrm{act}}(x,t), \tag{6}$$
$$\left\langle \lambda^{\mathrm{act}}(x,t) \right\rangle = 0, \tag{7}$$
$$\left\langle \lambda^{\mathrm{act}}(x,t)\lambda^{\mathrm{act}}(x',t') \right\rangle = -\sigma^2\,\tau\,\delta(t-t')\,a\,\delta^{(2)}(x-x'), \tag{8}$$
$$\delta^{(2)}(x) = \frac{\partial^2}{\partial x^2}\delta(x), \tag{9}$$

where x is our variable of space, t is the time, ζ is a drag coefficient, η is akin to a viscosity, B is akin to an elastic constant, λ^{act} is a Gaussian stochastic force, σ is an energy scale, τ is a time scale, a is a coarse-graining length scale, and δ is the Dirac delta function.

Q: What is the equal-time displacement spatial correlations $\langle u(x,t)u(x',t) \rangle$?